NEW RESULTS IN ASTRODYNAMICS USING GENETIC ALGORITHMS

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Genetic algorithms have gained popularity as an effective procedure for obtaining solutions to traditionally difficult space mission optimization problems. In this paper, a brief survey of the use of genetic algorithms to solve astrodynamics problems is presented and is followed by new results obtained from applying a Pareto genetic algorithm to the optimization of low-thrust interplanetary spacecraft missions . A hybrid optimization method was designed to integrate a Pareto genetic algorithm with a calculus-of-variations-based trajectory optimizer. Fronts of Pareto optimal trajectories were generated and novel trajectories identified for both Earth-Mars and Earth-Mercury missions.

INTRODUCTION

During the last several years, investigators have used genetic algorithms (GAs) to optimize various aspects of complex space missions. These missions are diverse in nature and range from trajectory planning for launch vehicles to the design of interplanetary missions. For example, a guidance method for the ascent trajectory of a single stage to low-earth-orbit (LEO) launch vehicle has been developed with the aid of a genetic algorithm.[1] The genetic algorithm was used to perform the off-line optimization of the nominal trajectory. The objective of the optimization was to maximize the payload delivered to LEO. A single constraint that enforced the dynamic pressure the vehicle experienced during the ascent was applied. Final results were compared with the high-fidelity optimization algorithm, Optimal Trajectories Implicit Simulation (OTIS) and found to be within 2% of the OTIS-predicted result. The resulting GA guidance method provided good performance, with computational costs lower than traditional methods.

Reference [2] investigated the feasibility of using a genetic algorithm for trajectory optimization of an ascent trajectory from the surface of the moon. The genetic algorithm generates a thrust profile for each trajectory with the overall objective to minimize the flight time for a given terminal orbit. The GA results were compared with an analytic solution and were also measured against results obtained by a calculus-based optimization method. Both comparisons were favorable. During the study, several local optima were found to exist. A hybrid methodology was developed where the GA is used to determine a nominal trajectory, and the calculus-based method then optimizes the GA-derived solution. The hybrid technique outperformed the GA alone. The study concluded that genetic algorithms are a feasible method for trajectory optimization.

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Another mission that GAs have aided in mission planning entails sending spacecraft to rendezvous with comets, land on their surface and take samples.[3] A GA was developed to determine a complex set of maneuvers to fly over five candidate landing sites. The objective was to minimize the propellant required by the chemical propulsion system during the fly-over of the five candidate landing sites. The maneuvering strategy was complicated by the existence of multiple nonlinear practical constraints. A gradient-based algorithm previously developed for the purpose of determining fly-over maneuvers was found to be incapable of dealing with the posed problem due to the number of fly-over sites and complicated constraints. The GA, however, was able to provide several satisfactory solutions.

Traditionally, interplanetary space missions have employed chemical high-thrust propulsion systems to perform the necessary energy exchanges that are required to complete the mission objectives. During preliminary design, the energy exchange is assumed to occur instantaneously so that the optimization process must determine the moment in time that the engines are activated, and the magnitude and direction of the velocity impulse imparted to the spacecraft. A one-way Earth to Mars trajectory was studied in Reference [4] and the associated parameter space was found to be multimodal and discontinous. The objective was to minimize the initial mass in LEO that would deliver a 6000 kilogram spacecraft to Mars. The Earth departure date and Mars arrival date were allowed to vary. A GA was successful in determining a global and several locally optimizing solutions.

The recent shift in NASA to a philosophy of better, faster, cheaper missions has lead to renewed interest in advanced propulsion systems. During October 1998, the first low-thrust propulsion interplanetary space mission, New Millennium Deep Space 1 (DS1), is scheduled to be launched. This new propulsion system requires a different dynamic model than the high-thrust counterpart due to the fact that the energy exchange imparted to the spacecraft occurs over a long period of time. Typically, low-thrust missions require the propulsion system to operate for a significant percentage of the entire mission length. The resulting optimization problem is complicated by the fact that the magnitude and direction of the propulsion system thrust is a continuous function of time. Mission objectives to be considered include minimizing the amount of propellant required to perform the mission or minimizing the entire mission length. References [5-6] used macro and micro GAs to determine the magnitude and direction at discrete locations along the trajectories. Mission constraints were enforced as penalties to the objective function. Results were shown to closely match published results that were determined using gradient-based optimization methods.

Genetic algorithms have also been a useful tool for determining near-optimal low-thrust spacecraft trajectories in instances when applying second-order necessary and sufficient conditions within a calculus of variations framework fail.[7-8] A simple GA coupled with an ordinary differential equation integrator for trajectory propagation was used. The GA determined the control parameters for a trajectory, and the system equations of motion are propagated forward using the integration routine to determine the final spacecraft state. Fitness was assigned according to the degree that the desired final state was met. Terminal constraints were applied via appended penalty functions.

The remainder of this paper describes an on-going joint research venture being conducted by the Jet Propulsion Laboratory (JPL) and the University of Illinois at Urbana-Champaign's Computational Astrodynamics Research Laboratory (UIUC CARL). A Pareto genetic algorithm is implemented to perform multi-objective optimization for low-thrust space missions, expanding the previous work of References [5-6] to better meet the needs of mission designers. Multi-objective optimization is the optimization of a system with more than one objective where the objectives may be competing. Development of the multi-objective optimization algorithm follows that described in Reference [9] and allows for the generation of fronts of optimal trajectories offering a variety of options for the mission planner. The Pareto genetic algorithm was combined with JPL trajectory optimization software and used to study an Earth-Mars and Earth-Mercury mission.

LOW-THRUST MISSIONS

In the last several years, there has been pressure to lower the costs of performing planetary missions. One way to accomplish less-expensive missions is to reduce overall mission length thereby reducing operational costs. Another method is to use on-board spacecraft propulsion systems that are highly efficient in their use of propellant. These propulsion systems use less propellant and therefore result in a less massive spacecraft. Smaller total spacecraft mass requires smaller launch vehicles to escape the Earth's gravitational well and may reduce total launch costs since the cost of launch vehicles increases with total payload mass launching capability. Solar electric propulsion (SEP) systems are one of the most efficient propulsion systems that are available for interplanetary missions. This makes SEP an attractive option for potentially achieving lower overall mission costs; however, SEP systems provide a thrust that is relatively small, typically on the order of fractions of Newtons. Low-thrust systems have thrusting profiles that are characterized by long periods of thrusting and the resulting trajectories can be difficult to optimize. This paper describes an approach to determine optimal trajectories for SEP systems that also provides the capability to minimize or maximize several objectives (i.e. flight time, propellant consumed, etc.) A Pareto GA is combined with JPL software to perform the multi-objective optimization.

PARETO OPTIMIZATION

Multi-objective optimization requires determining solutions to a system with more than one objective. As in single-objective optimization, the objective(s) may have any number of equality or inequality constraints imposed upon them. Equation (1) represents the multi-objective problem mathematically.

$$\begin{array}{lll} \text{Minimize/Maximize} & f_i(\mathbf{x}) & i = 1,2,\ldots,N \\ \text{Subject to} & g_j(\mathbf{x}) \leq 0 & j = 1,2,\ldots,F \\ & h_k(\mathbf{x}) = 0 & k = 1,2,\ldots,K \end{array} \tag{1}$$

Rather than searching for the solution which yields the globally maximal (or minimal) value for a single objective, the "best" solution is found by simultaneously optimizing several objectives at once. A common method for handling multiple objectives is to combine them into a single scalar objective by averaging each objective and then weighing its influence to the overall objective through a weighting factor. Such reduction techniques eliminate the need for a more complex multi-objective algorithm, but introduce new parameters in the form of weighting factors. Individual solutions are highly sensitive to the magnitudes of these weighting factors. The user must become familiar with the exact relationship between objectives in order to determine the proper weighting values that will yield the desired result. Improper assignment of weighting factors can result in a bias towards certain objectives.

In addition, the result of the optimization will be a single optimal solution rather than a set of optimal solutions that demonstrate the trades between different objectives. This is acceptable for cases in which optimality of all objectives coincide in the same solution, but in most cases where adjustments beneficial to one objective are detrimental to others, such a technique provides only a single point on what may be an expansive "front" of possible solutions. In the case of GAs, such a formulation fails to take advantage of the population-based nature of the technique. It therefore becomes desirable to develop a more robust, multi-objective algorithm, capable of identifying the relationships between objectives and able to make better use of the population-based GA to produce sets of optimal solutions.

PARETO GENETIC ALGORITHM

The GAs implemented here uses the standard operators of selection, cross-over and mutation. The algorithm devised in this study is based on the concept of nondominated sorting originally conceptualized by Goldberg[10] and developed by Srinivas and Deb[11] as the Nondominated Sorting Genetic Algorithm (NSGA). The NSGA uses the concept of nondominance to sort through a population of possible solutions, assigning each member to a Pareto optimal "front" according to their level of nondominance. The process consists of two iterated steps. The population is first sorted, and those individuals that are nondominated are assigned fitnesses initially equal to the size of the population, representing the maximum number of fronts possible. After assignment to a front, a technique known as "fitness sharing" or "niching"[6] is applied to these individuals. Niching adjusts the fitness of neighboring solutions in an attempt to evenly distribute individuals across the Pareto front. After niching is implemented, the minimal fitness in the current front is determined. This fitness is then slightly reduced and used as the initial fitness for the next front, and the nondominated sorting process continues until all individuals in the population are assigned fitness values.

The NSGA is not an entirely new algorithm, but rather a modification to the fitness evaluation procedures that exist in standard genetic algorithms. It is in some sense a supplement to the simple genetic algorithm (consisting of reproduction, cross-over and mutation) that allows for a more effective means of multi-objective optimization. The method may be viewed as an adaptation that filters the GA population immediately after undergoing fitness evaluation to reorient the population for a multi-objective optimization.

Testing the Algorithm

Verification of Performance

A set of test functions comprised of those found in the literature and of original design was used to test the NSGA. Six diagnostic test functions in all were used: three functions from the Srinivas and Deb test suite[11] to test replication of performance, along with three new functions designed to test previously undemonstrated capabilities required for the trajectory optimization work in this study. The algorithm developed by the authors, although based upon the original NSGA, is not a replica. Two differences are the substitution of the stochastic remainder selection scheme with stochastic universal selection and the use of a sharing scheme implementing normalized parameters. In order to facilitate comparison, values for main control parameters were assigned the same values as those used in ~~the~~ Reference [11].

Test Functions

The NSGA algorithm developed in this paper was tested against several test functions. First, the results using the three test functions considered in Reference [11] were reproduced and can be found in Reference [12]. These functions all involve two objectives. The first two possess one independent variable with no constraints, while the third utilizes two independent variables with two constraints. These three functions demonstrate the NSGA's ability to optimize functions with two objectives at most, each being strictly minimized; however the algorithm is capable of optimizing for any number of objectives, regardless of whether each individual objective is of minimizing or maximizing type.[11] Three additional test functions were devised to confirm these capabilities.

The first of these additional functions maintains two slightly out of phase sinusoids as objectives. The resulting function possesses four regions of Pareto optimality, and enables the examination of the algorithm's ability to maintain more than two subpopulations of Pareto optimal individuals. The problem is defined mathematically as,

Minimize/Maximize:

$$
\begin{aligned}
f_{11} &= 2\left\{\sin\left[2\pi\left(\frac{x}{5}-1\right)\right]+1\right\} \\
f_{12} &= 2\left\{\sin\left[2\pi\left(\frac{x}{5}\right)\right]+1\right\}
\end{aligned}
\tag{2}
$$

Furthermore, by varying which objective is maximized for optimality and which is minimized, the NSGA's ability to handle combinations of optimization types was investigated. The NSGA was successful at identifying the Pareto regions for all four permutations. Figure 1 demonstrates the results where f_{11} is maximized and f_{12} is minimized. For each value of x, the corresponding values for f_{11} and f_{12} are shown, where a circle is used for f_{11} and a diamond for f_{12}.

The second supplemental function has the purpose of verifying the NSGA's ability to handle more than two objectives. It was established that, for the purposes of the initial trajectory optimization study, no more than four objectives would be required and the following functions are designed to test this extended capability.

$$\text{Minimize:} \quad f_{21} = (x_1 - 2)^2 + (x_2 - 2)^2 \quad f_{22} = (x_1 + 2)^2 + (x_2 + 2)^2$$
$$f_{23} = (x_1 + 2)^2 + (x_2 - 2)^2 \quad f_{24} = (x_1 - 2)^2 + (x_2 + 2)^2 \tag{3}$$

This set of functions results in four parabolic objectives, whose Pareto optimal region lies within the square defined by their vertices. Figure 2 illustrates that the NSGA clearly identified the Pareto region.

The final test adds two constraints to the previous set of functions displayed in Equation (3). The two constraints applied were similar to those applied in test functions in Reference [11]. Individuals are constrained to reside within the circle and above the line given by:

$$\text{Constraints:} \quad (x_1 - 1.67)^2 + (x_2 - 0.5)^2 - 1 < 0$$
$$x_2 - 0.5x_1 > 0 \tag{4}$$

As in Reference [11], the constraints were applied via a penalty function appended to each objective. The algorithm possessed the ability to again correctly identify the Pareto optimal region, despite the added constraints as is shown in Figure 3. The control parameters used for each function are listed in Table 1. In each case, the normalized value of σ_{share} was determined based on the methodology outlined in Reference [13].

Table 1. Parameters for Test Function Pareto GA Runs

Control Parameter	*F*1	*F*2	*F*3
Maximum Generation	500	500	100
Population Size	100	100	100
String Length	28	30	30
Probability of crossover	1.0	1.0	1.0
Probability of mutation	0.0	0.0	0.0
σ_{share}	0.002	0.035	0.018
α	2.0	2.0	2.0
Number of parameters	1	2	2
Number of objectives	2	4	4
Number of constraints	0	0	2

HYBRID OPTIMIZATION METHOD: NSGA+SEPTOP

The algorithm formulated for determining optimal low-thrust trajectories combines the NSGA with the JPL calculus-of-variations-based optimizer, SEPTOP. The NSGA is used as a driver for the SEPTOP software which automates the optimization process by acting as a kind of "smart" user.

SEPTOP (Solar Electric Propulsion Trajectory Optimization Program) is a preliminary mission planning tool that uses a two-body, Sun-centered, low-thrust solar-electric propulsion model.[14] Classical Calculus-Of-Variations (COV) is used to obtain a maximum final mass resulting in a Two-Point-Boundary Value Problem (TPBVP). The user is required to provide initial estimates for costates (Lagrange multipliers) and the state and costate differential equations are integrated forward in time. Terminal constraints on the states and costates that result from the COV formulation must be satisfied. The convergence of SEPTOP to an optimal trajectory can be highly dependent on the user's initial guess and the relative difficulty of the mission. As the number of intermediate planetary flybys and total number of revolutions about the sun increases, the user's initial guess must move closer to their converged values for the TPBVP to successfully converge.

The NSGA evolves populations of individuals representing possible trajectories, with input parameters for SEPTOP being encoded as each individual's *genotype* (current values of the independent variables.) These input parameters include the costate values associated with a given trajectory, as well as the total time of flight. Individual fitness is evaluated through a call to SEPTOP using the input parameters encoded within. SEPTOP is run for a set number of iterations, effectively executing a localized search for each member in an attempt to better identify any basins of attraction that might exist in the individual's immediate vicinity. The improved fitness - if any improvement was seen - is returned to NSGA in the form of an objective vector containing the values for each objective in the multi-objective optimization. The individual is thus assigned a new *phenotype* (cost). The new SEPTOP input parameters associated with the improved solution are not returned as the individual's new parameter set, i.e. genotype. This is done in an attempt to maintain a greater amount of diversity in the population's gene pool.

Within this structure, the SEPTOP software is programmed to return the following four parameters; mass delivered, time of flight, number of heliocentric revolutions, and SEPTOP convergence error. Three of the parameters (mass delivered, time of flight, and number of heliocentric revolutions) are treated as objectives and the fourth (SEPTOP convergence error) as a constraint. The trade relationship between two of the objectives (mass delivered to the destination planet and time of flight.) is to be examined. A third criteria of maximizing heliocentric revolutions is employed as a strategy for obtaining viable solutions and is explained in detail in Reference [9]. Population members are also constrained through user-specification on the number of revolutions about the Sun. The two constraints are applied at the end of the evaluation procedure using appended penalty terms.

NEW RESULTS

Multiple interesting Earth-Mars and Earth-Mercury rendezvous missions have been determined using the hybrid optimization tool. The control parameters used in each case are listed in Table 2. Selection and crossover operators were implemented with no mutation as was done for the test functions. Niching parameters were calculated based upon the methods provided Reference [13]. The induced number of niches was set to 15 -- the same value proportional to population size as in the test functions. Existing methodologies for establishing σ_{share} are only rough guides however, since they require knowledge of the search space for each optimization problem a priori. Guidance for population sizing and maximum generation determination for hybridized methods was found to be nonexistent at the time of this study, therefore population sizing and number of generations were determined mainly based on computer processor limitations. A mid-range performing launch vehicle, the Delta II 7925, was used for both test cases along with a single 30 cm xenon engine for spacecraft propulsion. A 5.2 kW solar array was used for the Mars mission and a 5.0 kW array for the Mercury mission. The output reference power is that produced by the arrays at one astronomical unit, the mean distance of the Earth from the Sun.

Table 2. Parameters for Pareto Trajectory Optimizations

Control Parameter	Parameter Value
Population Size	150
String Length	80
Probability of crossover	1.0
Probability of mutation	0.0
σ_{share}	0.033
α	2.0
Number of parameters	8
Number of objectives	3
Number of constraints	2

Earth-Mars Rendezvous

The first set of results is for Earth-Mars rendezvous trajectories. Figure 4 illustrates the population of converged Pareto optimal trajectories at generation 30. In this case, 91 of the 150 population members are nondominated. Trajectories were restricted to 0.5 to 3.0 heliocentric revolutions, flight time bound between approximately 10 months and 3.5 years, and launch date again fixed at September 1, 2005. Converged Pareto optimal trajectories are indicated with an ×. Solutions with trajectory plots that will be described later are labeled by individual number.

Two distinct groupings of Pareto optimal individuals were identified and are defined by curves in Figure 4. Figure 5 reveals the largest and most evenly distributed family in the

Pareto space - bound by individuals 44 and 36 - to be the largely dominating subgroup. Beginning with individual 44 (Fig. 6) from the largest family with a delivery mass of 685.28 kg, performance increases rapidly with increasing transfer time. This performance plateaus as a coast arc appears in the trajectories neighboring individual 73, and delivery masses reach approximately 862 kg. As is the case for all trajectory plots to follow, solid line segments represent thrusting arcs, and dashed line segments coast arcs. Only very small improvements in mass delivered are seen on the section of the curve between individuals 73 and 72. A second coast arc appears producing a burn-coast-burn-coast sequence for these solutions. In the vicinity of individual 72, an additional burn is appended creating burn-coast-burn-coast-burn trajectories and performance again rises. Performance for this subgroup of solutions approaches its maximum as trajectories transition back to a burn-coast-burn structure at individual 36 with a mass delivered of 884.10.

The small front of solutions whose population resides between individuals 132 and 22 is also worthy of discussion due to the high performance and novel trajectory structure (Fig. 7) that are characteristic of its members. These solutions begin by taking an inward direction and spend some time performing heliocentric revolutions within or just outside of Earth orbit before spiraling out to Mars. In these subjects, two revolutions are made to increase the spacecraft's orbit inclination to more closely match that of Mars' before the trajectory progresses outward. A significant increase in performance is seen as flight times increase. The minimum mass delivered for this subset of the population is 773.33 kg (individual 132) with a maximum of 911.78 (individual 22) - the highest payload delivery in the population.

Earth-Mars Rendezvous On-Demand

The high performance obtained by members of this Pareto family of novel solutions prompted further investigation into the potential of such a trajectory class. The control parameters for the trajectory with the highest performance (individual 22 with 911.78 kg) were extracted, and the SEPTOP software run to compute the performance over a range of launch dates spanning approximately one Martian synodic period. A synodic period is the length of time that must elapse before the relative geometry between two planets repeats and is 778 days for the planets Earth and Mars. For purposes of comparison, the same control parameters were also used to generate series of trajectories over the same range of launch dates with transfer times of 1.5, 2.5, and 3.0 years.

Results of this analysis are summarized in Fig. 8. The dashed curves in the figure represent multiple revolution SEPTOP solutions (2 and 3 revolutions) possessing flight times ranging from 2.5 to 3.55 years. The solid curve provides a comparison with a more typical SEPTOP solution: 1.5 years and less than 1 heliocentric revolution. These curves reveal a continuous period of launch dates for a flight time of 3.55 years, all with final spacecraft mass greater than 900 kg. Shorter flight times with very large (but not continuous) launch periods are also available, such as the 2.5 year curve which has a launch period close to one year with performance greater than 900 kg.

Earth-Mercury Rendezvous

The second case considered involves an Earth-Mercury rendezvous trajectory with multiple revolutions. Desired trajectories were constrained to be between 3.5 to 6.0 heliocentric revolutions and the launch date was set at May 31, 2003. Using a population of 150, 62 solutions were determined. The solutions broke into two major fronts and can be seen in Figure 9. One front is bound by individuals 15 and 45 and the second by individuals 38 and 19. Figure 10 illustrates the trajectories delivering the greatest payload to Mercury for the two fronts. For individual 19, the initial mass of the spacecraft is 968 kg and the delivered mass is 584 kg. Individual 45 has an initial mass of 765 kg and delivers a spacecraft mass of 484. The Pareto regions identified in this trial are more sparsely populated than in the previous case due to the increased difficulty of the mission. Allowing the hybrid algorithm to run for additional generations (30 were used in this run) might aid to fill out the fronts.

Earth-Mercury Rendezvous Availability

no fig. 11 included

The two higher performing individuals (19 and 45) in each front were investigated further to determine if these trajectories could produce good performance for large ranges of launch dates. The period of interest is an Earth-Mercury synodic period, which is 116 days. Figure 11 displays the results for four different cases. Two are based on individual 19 but with total mission lengths of 2.25 and 2.0 years and two are based on individual 45 but with flight times of 1.75 and 2.0 years. The performance of the cases based on individual 45 was similar for the launch dates between mid-June to mid-July. For launch dates proceeding these few months, the 2.0 year flight time outperforms the 1.75 year trajectories. A similar trend is found for the two cases based on individual 19 where there are launch dates where the performance of both flight times only varies by a few kilograms and these dates are proceeded by months where the 2.25 year trajectories outperform the 2.0 year missions. An interesting observation is that by combining the results of individual 45 and 19 for the 2.0 year flight time, a relatively flat performance is achieved over the entire synodic period. Also, individual 19 offers higher consistent performance with the 2.25 year flight time but at the cost of a longer missions. By identifying multiple solutions, the hybrid technique has produced missions that are readily available.

CONCLUSIONS

A Pareto genetic algorithm was combined with a calculus-of-variations optimization algorithm and proved an effective method for generating sets of optimal interplanetary trajectories. Fronts of optimal trajectories were generated for an Earth-Mars and Earth-Mercury mission. In both cases, novel trajectories were found by the hybrid method and offered interesting options to the mission designer.

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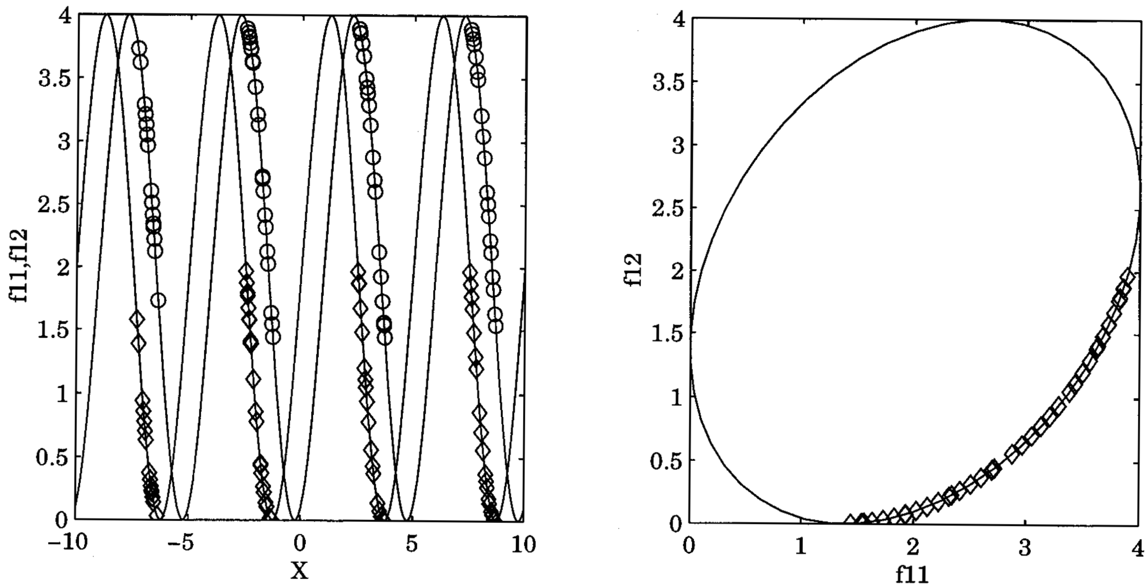

Figure 1. Test Function $F1$ at Generation 500

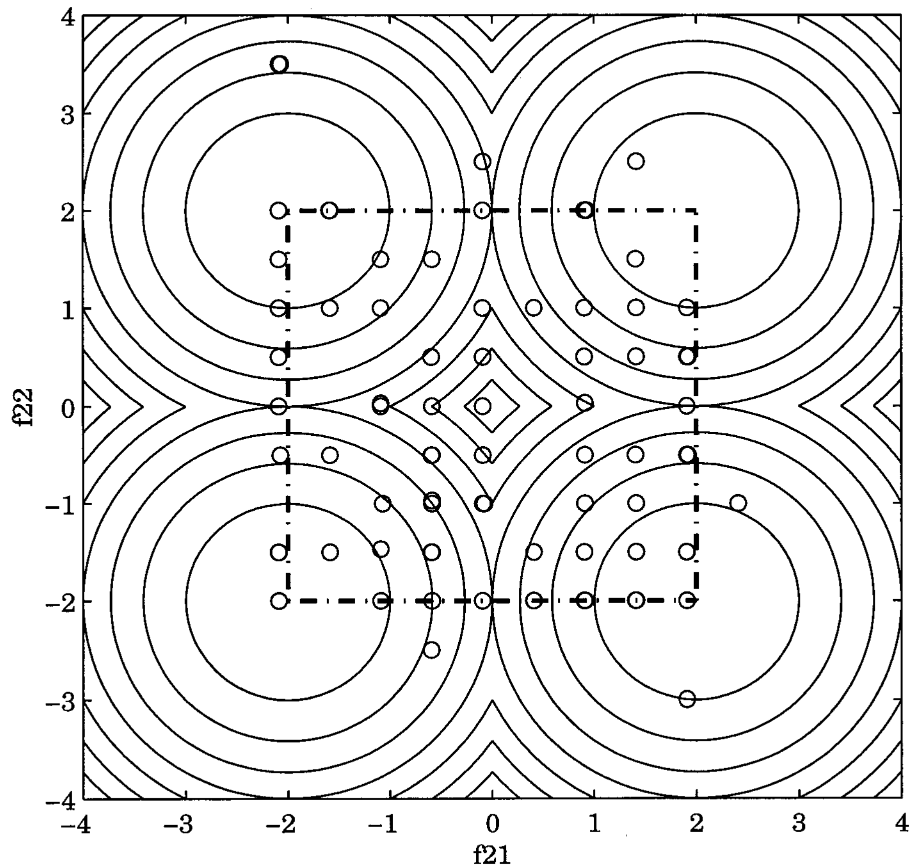

Figure 2. Test Function *F*2 at Generation 500

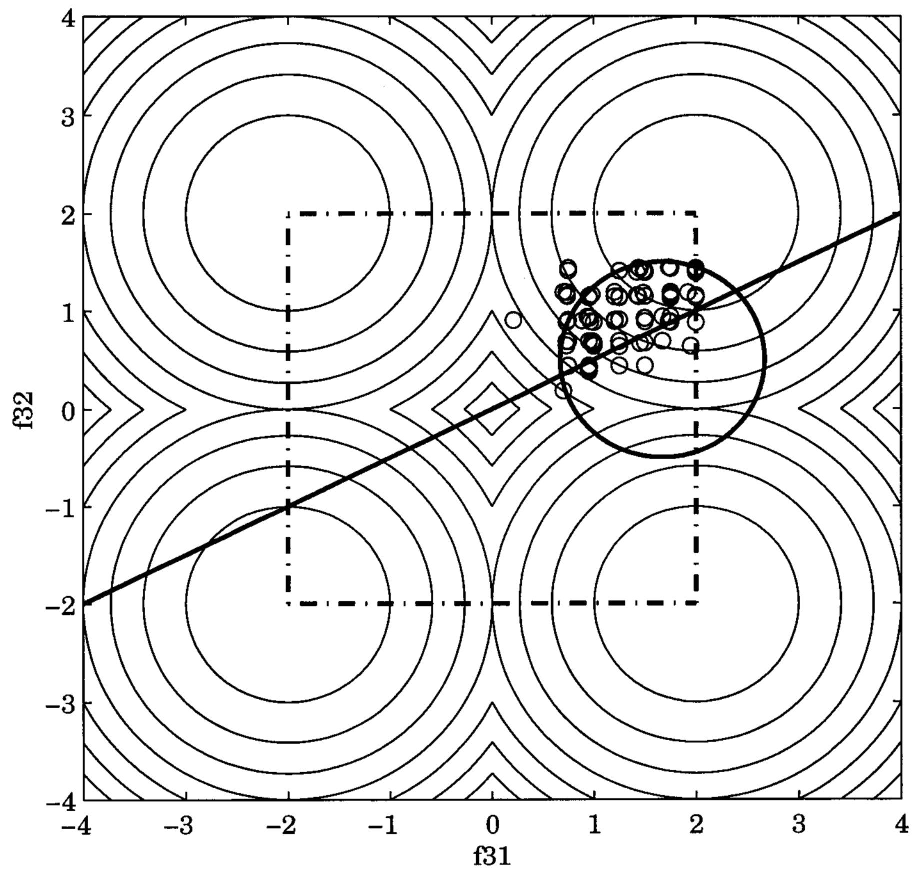

Figure 3. Test Function $F3$ at Generation 100

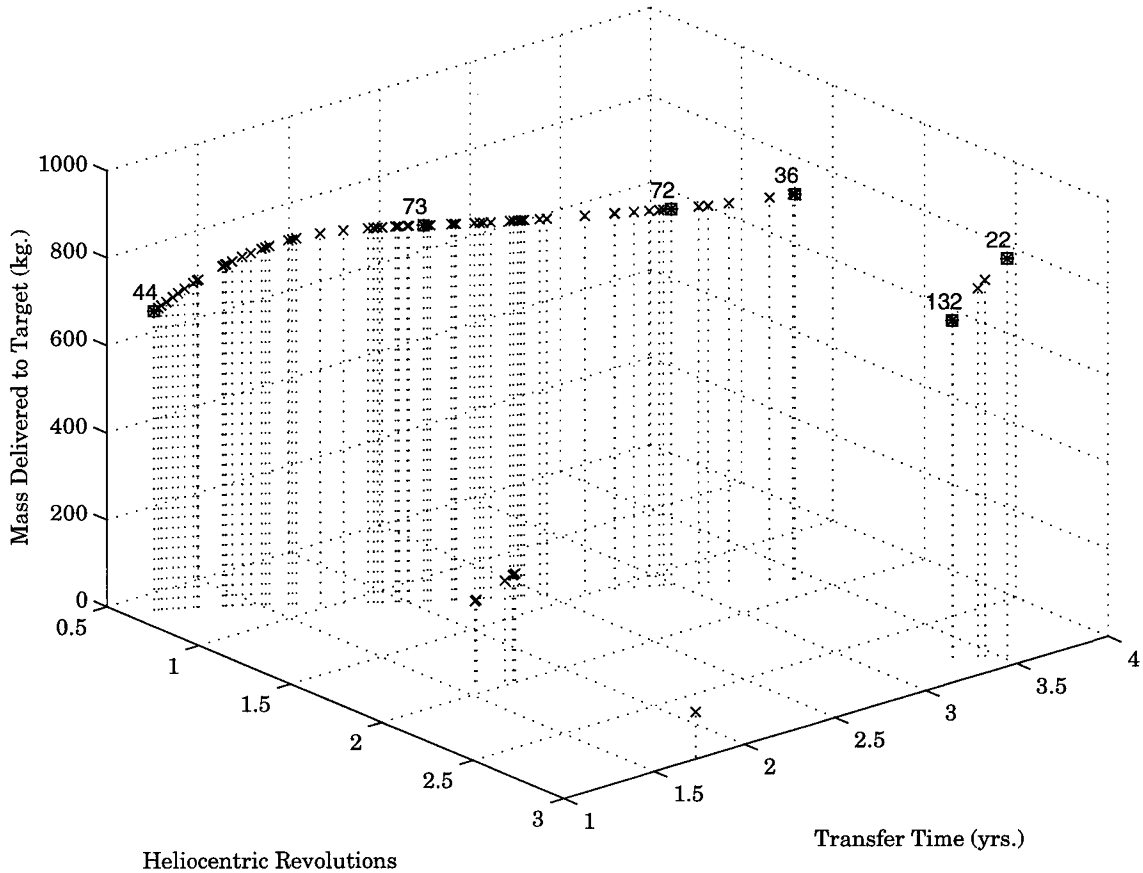

Figure 4. Fronts of Pareto Optimal Solutions For Earth-Mars Rendezvous

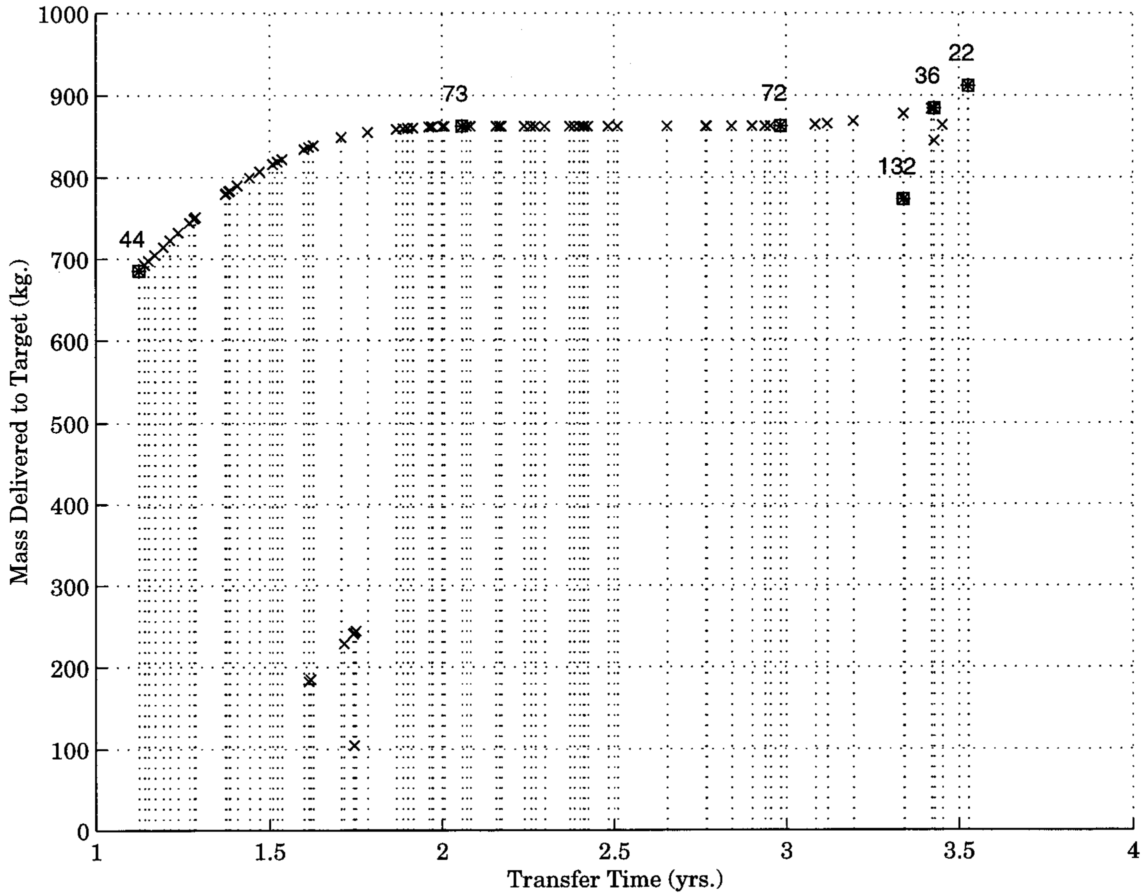

Figure 5. Mass Delivered to Mars Versus Transfer Time

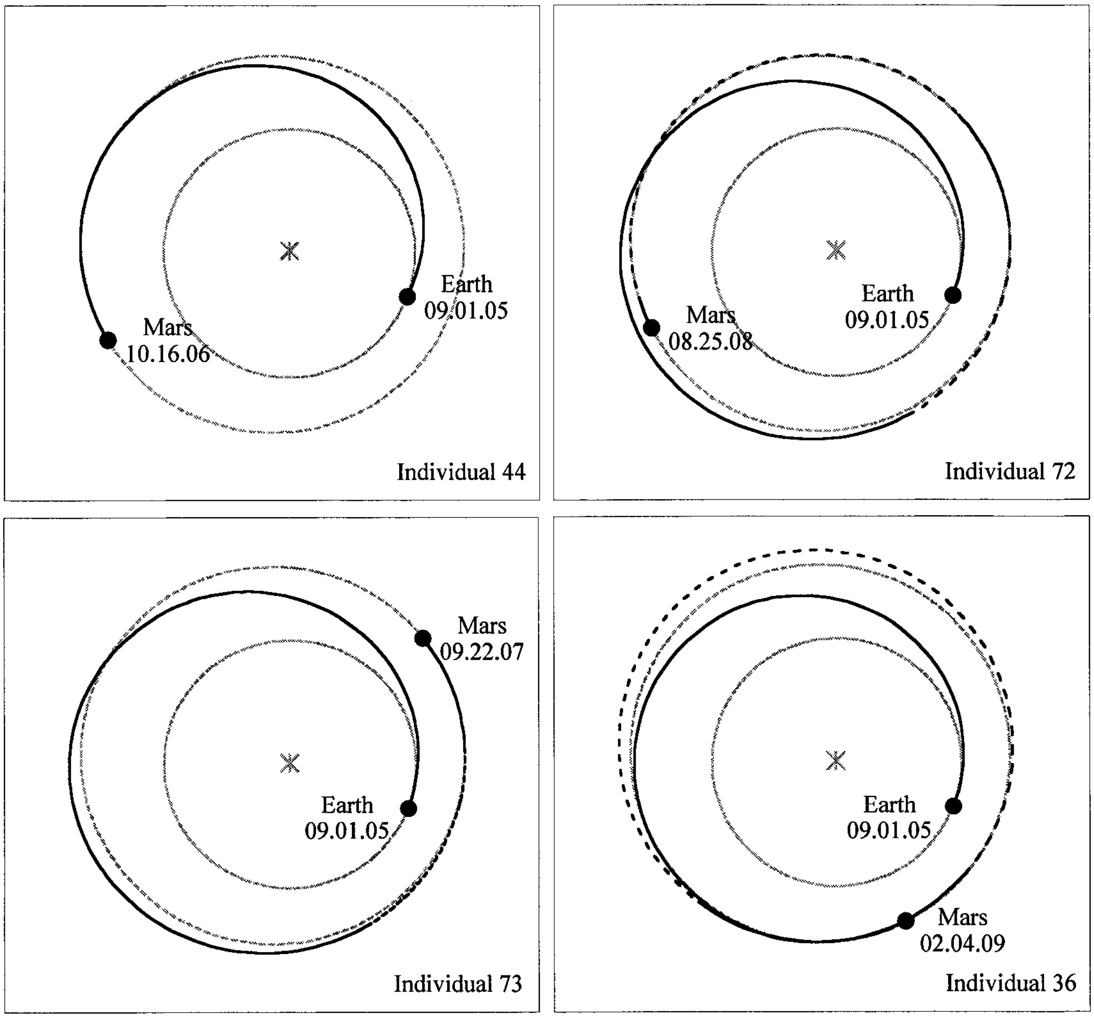

Figure 6. Earth-Mars Trajectories for Select Nondominated Individuals

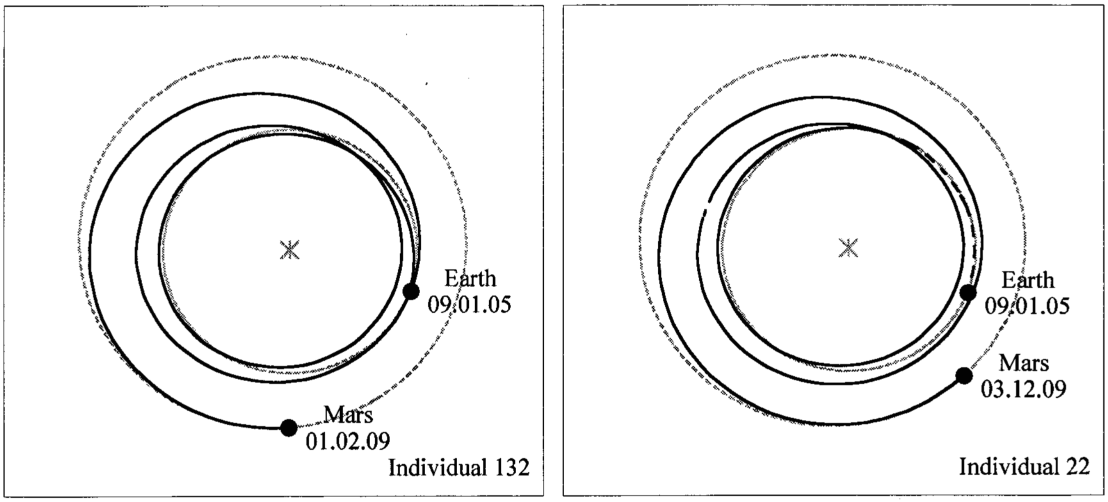

Figure 7. Earth-Mars Trajectories With Novel Performance Feature

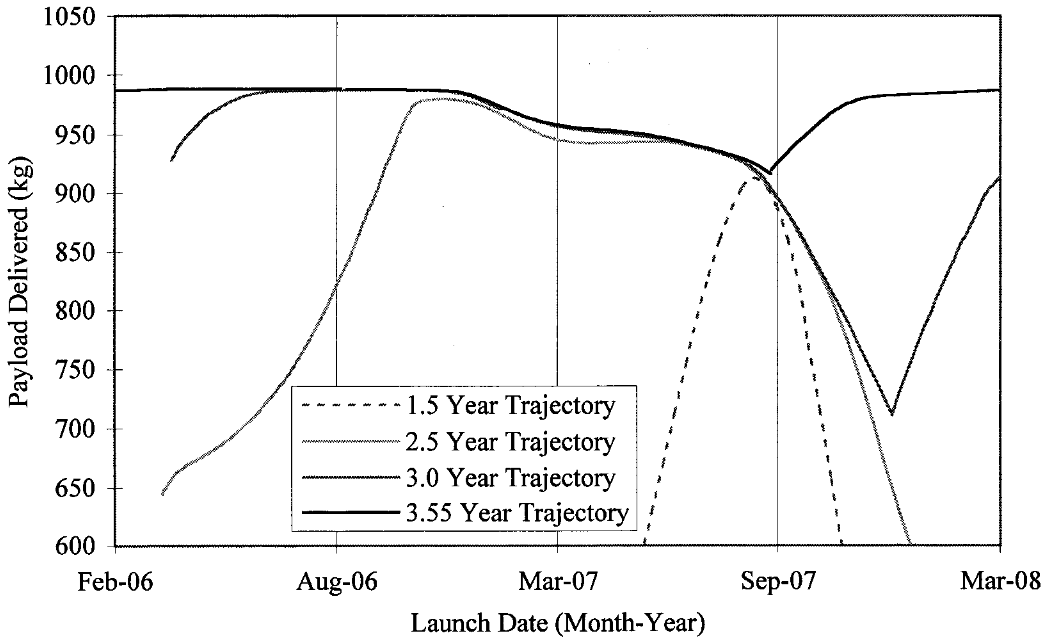

Figure 8. Performance of Earth-Mars Rendezvous Versus Flight Time

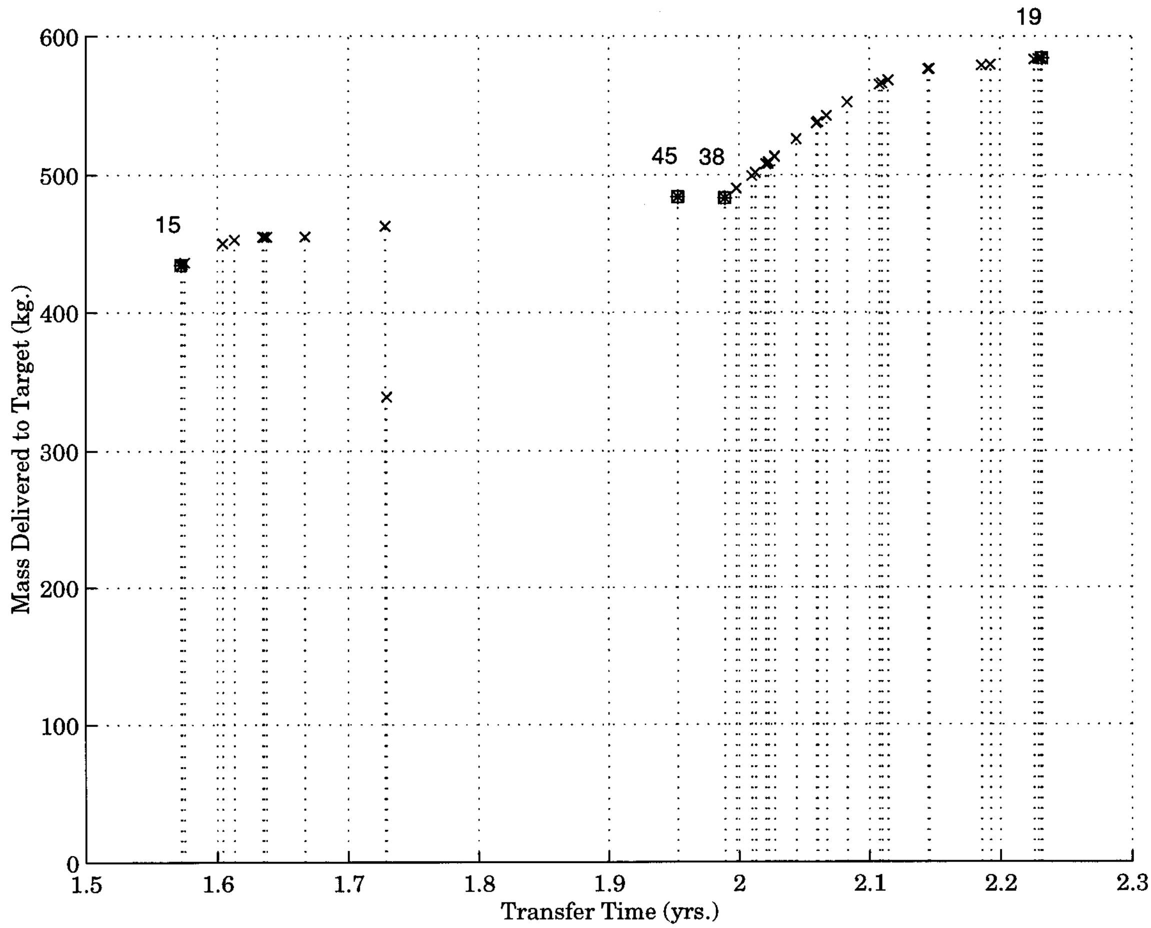

Figure 9. Mass Delivered to Mercury Versus Transfer Time

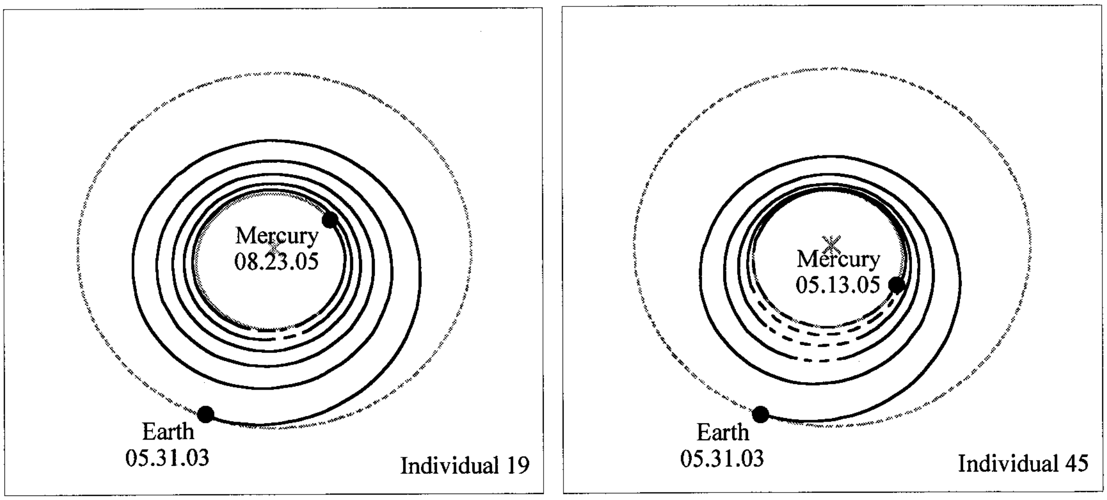

Figure 10. Earth-Mercury Trajectories for Select Individuals